Effect of thermal fluctuations of spin density wave order parameter on the quasiparticle spectral function

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The two-dimensional model of itinerant electrons coupled to an antiferromagnetic order parameter is considered. In the mean-field solution the Fermi surface undergoes reconstruction and breaks into disconnected "pockets". We have studied the effect of the thermal fluctuations of the order parameter on the spectral density in such system. These fluctuations lead to a finite width of the spectral line scaling linearly with temperature. Due to the thermal fluctuations the quasiparticle spectral weight is transferred into a magnetic Brillouin zone. This can be interpreted as restoration of "arcs" of the noninteracting Fermi surface.

I. INTRODUCTION

Fluctuations play a prominent role in systems of reduced dimensionalities leading to a complete or partial suppression of long-range order and rendering mean-field approximation inapplicable. Calculation of correlation functions then becomes an arduous task. The question is whether in the absence of true long-range order these functions display features of the ordered state and if yes, to what extent. The correlation function we are concerned with in this paper is the single-electron spectral function. Measurements of this function constitute the most powerful experimental probes in the physics of strongly correlated systems. We discuss the situation when the system is close to being antiferromagnetically ordered and study the effect of thermal order-parameter fluctuations.

There is a considerable literature addressing the influence of quantum fluctuations (we refer the reader to Refs. 1 and 2 which also provide references to the related papers). However, at finite temperatures one has to take into account thermal fluctuations which bring specific problems. In our previous publication[3] we considered the influence of thermal fluctuations on the spectral function of a two-dimensional (2D) superconductor. 2D superconductors have quasilong-range order such that phase fluctuations are critical in the entire temperature region below T_c. We have found that at least as far as the thermal fluctuations were concerned, the frequently used approach based on the conversion of this problem into a gauge theory turned out to be inadequate. Indeed, the power of the approach rests on the fact that to estimate the effect of fluctuations one needs to know only their Ginzburg-Landau free energy which is universal and is constrained only by the symmetry of the order parameter. However, this advantage is lost if one uses the procedures employed in the gauge theory approach. The latter approach uses a gauge transformation of the fermion fields with a subsequent attempts to treat the problem as a gauge field theory one (as, for instance, in Ref. 2). Such transformation, however, results in the diagram series where individual diagrams contain severe ultraviolet divergencies. Hence the universality is lost. As an alternative we have suggested the direct perturbation expansion in the order parameter. This procedure preserves universality since the dominant contribution to all diagrams comes from large distances.

In the present paper we apply the approach of Ref. 3 to calculate the spectral function in the presence of a fluctuating commensurate spin density wave (SDW). This problem has a potential relevance to the problem of cuprates. There is a significant experimental evidence suggesting that the Fermi surface of the cuprates in the underdoped regime undergoes reconstruction (see, for example, Refs. 4 and 5). It is frequently suggested that this reconstruction has a magnetic origin.[6,7] On the other hand, it is still unclear whether one needs a real long-range order to observe such reconstruction or a short range one will suffice. In the present paper we will address this question in the context of the spectral function. We consider only classical (thermal) fluctuations of the order parameter. This places us in what is called renormalized classical regime.

II. DESCRIPTION OF THE MODEL AND THE RESULTS

We consider a popular spin-fermion model in two dimensions where electrons interact with a commensurate SDW.[1] The antiferromagnetic ordering open gaps at the points of the Fermi surface (FS) connected by the vector of antiferromagnetic fluctuation $\mathbf{Q}=(\pi,\pi)$, see Fig. 1. As a result the FS
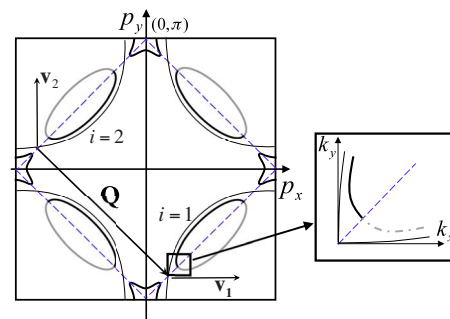

FIG. 1. (Color online) Formation of a Fermi pockets (thick solid line) in the mean-field Fermi surface reconstruction caused by the SDW ordering. The bare Fermi surface (thin solid line) is not nested. The dashed line is the magnetic Brillouin-zone boundary. Two subbands with $i=1,2$ are connected by the antiferromagnetic wave vector $\mathbf{Q}=(\pi,\pi)$. Fermi velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are assumed to be perpendicular. Inset shows two subbands brought together.

undergoes reconstruction into disconnected pockets. The Hamiltonian for quasiparticles located near two FS points connected by $\boldsymbol{Q}$ is

$$H = \sum_{k\alpha} \xi(\boldsymbol{k}) \psi_{k\alpha}^\dagger \psi_{k\alpha} + J \sum_k \boldsymbol{S} \psi_{k+Q\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} \psi_{k\beta}, \tag{1}$$

where $\boldsymbol{k}$ is a momentum vector in the Brillouin zone and α is the spin index. The kinetic energy close to the "hot spots" can be approximated as

$$\sum_{k\alpha} \xi(\boldsymbol{k}) \psi_{k\alpha}^\dagger \psi_{k\alpha} \approx \sum_{i=1,2} \sum_{k\alpha} \boldsymbol{v}_i \boldsymbol{k} \psi_{ik\alpha}^\dagger \psi_{ik\alpha} \tag{2}$$

with the index i enumerating two subbands related by the vector $\boldsymbol{Q}$. The sum in Eq. (2) is over small momenta and two subbands can be combined together (see inset in Fig. 1). We assume that there is no nesting; to simplify the calculations we consider the case when the corresponding Fermi velocities are perpendicular to each other: $\boldsymbol{v}_1 = v\hat{x}$ and $\boldsymbol{v}_2 = v\hat{y}$. At the mean-field level the spectrum is determined by the equation

$$\omega^2 - v(k_x + k_y)\omega + v^2 k_x k_y - J^2 = 0 \tag{3}$$

with the solutions

$$\omega_{1,2} = v(k_x + k_y)/2 \pm \sqrt{[v(k_x - k_y)/2]^2 + J^2} \tag{4}$$

signifying tips of electronlike and holelike Fermi pockets.

Fluctuations of the SDW order parameter transform rigid energy gaps into pseudogaps and smear the sharp peaks in the spectral function. Below we will study this process in detail.

It has been demonstrated in Ref. 1 that the feedback from the quasiparticles onto the spin Hamiltonian makes significant changes in the spin dynamics but does not affect zero-frequency modes. Since we consider only classical (thermal) fluctuations, such feedback will be neglected.

For the sake of simplicity we also assume that the vector of spin polarization lies either (i) in the XY plane or (ii) directed along the z axis. In both cases the transition occurs at finite temperature. In the first case the order parameter has U(1) symmetry and the transition is of the Kosterlitz-Thouless type. The spin fluctuations below T_c are critical. This power-law behavior will also hold above T_c, though only at distances smaller than the correlation length ξ. However, since the latter length is exponentially large in $(T-T_c)$, there is a range of temperatures $T > T_c$ and energies where the obtained expressions for the spectral function remain valid. In all this region where the magnetic correlation length is either infinite $(T < T_c)$ or exponentially large, the order parameter fluctuations can be considered as classical. This is essential for our approach. In the second case the order parameter has Z_2 symmetry and below T_c it acquires finite expectation value. Hence $T < T_c$ region is trivial and we will be concerned only with $T \geq T_c$ region. The correlation length in this region is $\xi \sim (T-T_c)^{-1}$; to neglect quantum fluctuations we need it to be much larger than T^{-1} meaning that we need to stay close to T_c.

We start with the easy-plane anisotropy case; the easy axis case can be obtained as a simple generalization. The fluctuating order parameter is staggered magnetization $\boldsymbol{S}$, it lies in

a plane and forms an angle ϕ with the fixed direction in the plane. Under the assumptions described above the quasiparticle Lagrangian is simplified

$$\mathcal{L} = \sum_{i,\alpha} \bar{\psi}_{i,\alpha}[\partial_\tau + \xi_i(-i\boldsymbol{\nabla})]\psi_{i,\alpha} + J\sum_{\alpha\beta} e^{i\phi}\bar{\psi}_{1,\alpha}\sigma_{\alpha\beta}^-\psi_{2,\beta} + \text{c.c.}, \tag{5}$$

where $\xi_i(\boldsymbol{k}) = \boldsymbol{v}_i \boldsymbol{k}$. We assume that the free energy for the classical phase field ϕ is Gaussian

$$\frac{F}{T} = \frac{\rho_s}{2T} \int dx dy [(\partial_x \phi)^2 + (\partial_y \phi)^2]. \tag{6}$$

Now the problem looks similar to the one of the thermal fluctuations in superconductors considered in our previous paper.[3] We stress that Eq. (6) is an effective free energy, where the stiffness ρ_s is renormalized by the processes involving, e.g., particle-hole excitations. In addition, the unharmonic terms in the free energy, Eq. (6), involve higher gradients rendering them irrelevant for the present calculation.

The previous attempts to describe the fluctuation effects in renormalized classical regime dealt with the isotropic case.[8,9] Instead of summing the diagram series these papers used semiqualitative approach. The electron self-energy was represented by the first-skeleton diagram with the Ornstein-Zernike propagator for the spin field and with the vertex replaced by a phenomenological constant. Another approach based on $D-2$ expansion has been recently employed in Ref. 10.

To be definite we consider the propagator of the spin-up particles and sum up leading contributions from all diagrams. Here we advantage of the fact that multipoint correlation functions of bosonic exponents in the Gaussian model in Eq. (6) are known explicitly. The spectral weight reaches its maximal value in the vicinity of the bare mass shell, $\omega \sim vk_x$ and also close to the mass shell of the spin-down particle $\omega \sim vk_y$ (the shadow mass shell). These two regions form two complementary parts of the Fermi pocket. As the spectral weight is small at the magnetic Brillouin-zone boundary, $k_x \sim k_y$ we have studied the Green's function separately at $\omega \sim vk_x$ and $\omega \sim vk_y$. In what follows we set the Fermi velocity to one, $v = 1$.

The summary of our results is as follows. At the mass shell we got the following expression for the Green's function:

$$G^{-1} = G_{\text{mf}}^{-1} + \frac{2da^{4d}\Gamma^2(2-2d)J^4}{[-i(\omega-k_y)]^{4-4d}} G_{\text{mf}}^{-1} \ln\left(\frac{G_{\text{mf}}^{-1}}{\omega-k_y}\right)$$
$$+ \frac{2dia^{6d}\Gamma^2(2-2d)\Gamma(1-2d)J^6}{[-i(\omega-k_y)]^{5-6d}}, \tag{7}$$

where $d = T/4\pi\rho_s$ is the scaling dimension of the order parameter, a is the lattice constant, and the mean-field Green's function is

$$G_{\text{mf}}^{-1} = \omega - k_x + \frac{iJ^2 a^{2d}\Gamma(1-2d)}{[-i(\omega-k_y)]^{1-2d}}. \tag{8}$$

We would like to emphasize that the expression (7) does not rely on the smallness of the parameter $d < 1/2$.

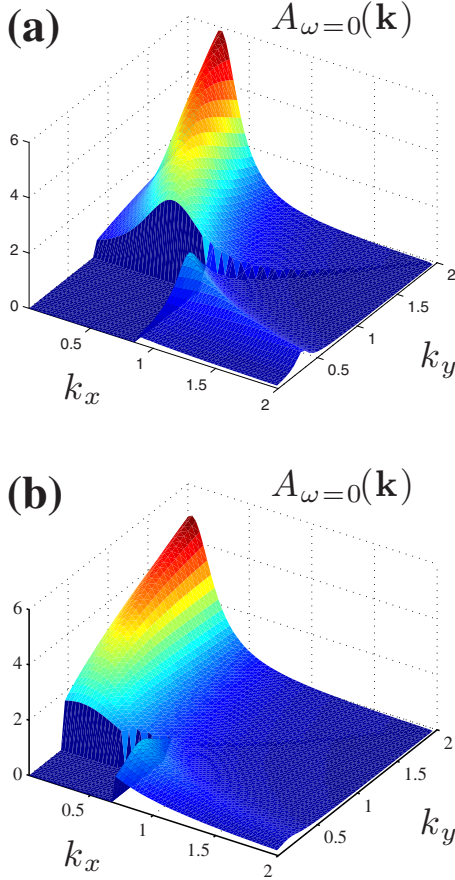**(a)** $A_{\omega=0}(\mathbf{k})$

(b) $A_{\omega=0}(\mathbf{k})$

FIG. 2. (Color online) The spectral density at the Fermi surface, $A_{\omega=0}(\mathbf{k})$, as given by Eqs. (7) and (9) at $k_y > k_x$ and $k_x > k_y$, respectively. These two graphs are separated by the area where the presented derivation ceases to be valid. The parameter d is (a) $d = 0.08$ and (b) $d = 0.2$.

At the shadow mass shell, $\omega \sim k_y$ we get

$$G = \frac{J^2 e^{i\pi d}\Gamma(1-2d)}{(\omega-k_x)^2}\left\{\omega-k_y+\frac{iJ^2a^{2d}\Gamma(1-2d)}{[-i(\omega-k_x)]^{1-2d}}\right\}^{-1+2d}. \qquad (9)$$

Our analytical results, are presented graphically in Fig. 2. The area of validity of these expressions is controlled by the energy scale

$$T_K = J(Ja)^{d/(1-d)}. \qquad (10)$$

Equation (7) is valid for $|k_y| > T_K$. Equation (9) is valid for $|k_x| > T_K$.

The remaining part of the paper contains a derivation of the results, Eqs. (7) and (9). Following the approach of Ref. 3 we develop a perturbation theory in the coupling constant J. Though this perturbation theory is free of ultraviolet singularities it contains infrared singularities at $\omega \sim k_{x(y)}$ which we sum up.
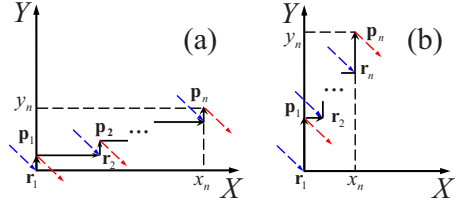

FIG. 3. (Color online) Graphical representation of the self-energy correction of the order $2n$ in coupling constant, Eq. (16). Solid vertical and horizontal lines represent segments of a real-space electron trajectory for (a) particle close to the mass shell, $\omega \approx k_x$ and (b) particle close to the shadow mass shell, $\omega \approx k_y$. Incoming (blue) and outgoing (red) arrowed skew dashed lines represent exponential factors $e^{i\phi(\mathbf{r}_i)}$ and $e^{-i\phi(\mathbf{p}_j)}$, respectively.

III. BEHAVIOR AT THE MASS SHELL: $\omega \sim k_x$

In this section we derive the result in Eq. (7). It is useful to consider the self-energy $\Sigma_\omega(\mathbf{k})$ defined in the standard way by the Dyson equation

$$G_\omega(\mathbf{k}) = [\omega - k_x - \Sigma_\omega(\mathbf{k})]^{-1}. \qquad (11)$$

As is shown below, the self-energy is a regular function of frequency at the mass shell and only weakly logarithmically nonanalytic in the coupling constant.

The J^2 contribution to the self-energy is

$$\Sigma_\omega^{(2)}(\mathbf{k}) = \frac{-i(Ja^d)^2\Gamma(1-2d)}{[-i(\omega-k_y)]^{1-2d}}. \qquad (12)$$

We notice that in the limit of infinite phase stiffness, $d=0$, Eq. (12) reproduces the mean field spectrum, Eq. (4), as expected. Note that the self-energy Eq. (12) is regular at the mass shell. This, however, is not the case for higher-order contribution. In fact, the fourth-order contribution has a weak logarithmic singularity (see Appendix A 1)

$$\Sigma^{(4)} = 2d\frac{-i(Ja^d)^4\Gamma^2(2-2d)}{[-i(\omega-k_y)]^{3-4d}}\alpha\log\alpha, \qquad (13)$$

where

$$\alpha = \frac{\omega+i0-k_x}{\omega+i0-k_y}. \qquad (14)$$

The aforementioned analyticity of the self-energy at the mass shell is restored once the leading on-shell singularities in all orders in J^2 are summed up. The reminder of the present section is devoted to this task.

Using the expressions for the bare (retarded) Green's functions

$$iG_{1(2)}^{(0)}(\omega,\mathbf{r}) = \theta(r_{x(y)})\delta(r_{y(x)})e^{i\omega r_{x(y)}}. \qquad (15)$$

we write the self-energy at the order J^{2n} with $n \geq 3$ in the form (see Fig. 3)

$$\Sigma_\omega^{(2n)}(\mathbf{k}) = i(-iJ)^{2n} \int_0^\infty dx_n e^{i(\omega-k_x)x_n} \int_0^\infty dy_n e^{i(\omega-k_y)y_n}$$

$$\times \prod_{i=2}^{n-1} \int_0^{x_{i+1}} dx_i \prod_{i=1}^{n-1} \int_0^{y_{i+1}} dy_i$$

$$\times C^{(2n)}(\mathbf{r}_1, \dots, \mathbf{r}_n; \mathbf{p}_1, \dots, \mathbf{p}_n), \qquad (16)$$

where $\mathbf{r}_i = (x_i, y_{i-1})$, $\mathbf{p}_i = (x_i, y_i)$, and $x_1 = y_0 = 0$, (see Fig. 3) and the cumulant in the last line

$$C^{(2n)}(\mathbf{r}_1, \dots, \mathbf{r}_n; \mathbf{p}_1, \dots, \mathbf{p}_n) = \delta_{1,n} - \sum_{i=1}^{n-1} \delta_{1,i}\delta_{i+1,n} + \cdots$$

$$+ (-1)^n \delta_{1,1} \cdots \delta_{n,n} \qquad (17)$$

is expressed in terms of averages of the exponents of the phase fields

$$\delta_{i,i+l} = \langle e^{i\phi(\mathbf{r}_i)+\cdots+i\phi(\mathbf{r}_{i+l})} e^{-i\phi(\mathbf{p}_i)-\cdots-i\phi(\mathbf{p}_{i+l})} \rangle. \qquad (18)$$

The latter average with free energy, Eq. (6) is well known

$$\delta_{1,n} = a^{2dn(n-2)} \left\{ \frac{\tilde{\prod}_{i,j=1}^n |\mathbf{r}_i - \mathbf{r}_j| \tilde{\prod}_{i,j=1}^n |\mathbf{p}_i - \mathbf{p}_j|}{\prod\limits_{i,j=1}^n |\mathbf{r}_i - \mathbf{p}_j|} \right\}^{2d}, \qquad (19)$$

where tilde over the product sign excludes $i=j$.

In what follows we sum up the most singular terms in expansion in Eq. (16). Our solution is based on the physical idea that at the mass shell particle propagates over long distances before it gets scattered by a thermal fluctuation. As in our case the on mass shell particle's direction of motion is along X axis, see Fig. 3(a), the horizontal segments in the staircase shaped real-space trajectory in Fig. 3(a) are longer than the vertical ones by a factor of $(\omega - k_y)/(\omega - k_x) \gg 1$. In other words, the mass shell singularity comes from the integration region $y_i \ll x_j$. Accordingly, we introduce new variables ξ_i

$$y_n - y_{n-1} = \xi_n x_n, \dots, y_1 - y_0 = \xi_1 x_n \qquad (20)$$

and argue that the important domain of integration is $\xi_i \ll 1$. We expand the cumulant in Eq. (17) in powers of ξ_is and retain the lowest power term to get the most singular contribution. It can be shown by inspection that this expansion gives

$$C^{(2n)} \approx a^{2dn} \prod_{i=1}^n (y_i - y_{i-1})^{-2d}$$

$$\times \left\{ \frac{[x_n^2 + (y_{n-1} - y_0)^2]^d [x_n^2 + (y_n - y_1)^2]^d}{[x_n^2 + (y_n - y_0)^2]^d [x_n^2 + (y_{n-1} - y_1)^2]^d} - 1 \right\}$$

$$\approx -2da^{2dn}(x_n)^n \xi_1 \xi_n \prod_{i=1}^n \xi_i^{-2d}. \qquad (21)$$

We substitute Eqs. (20) and (21) in Eq. (16), and perform integrations over x_n. We now analyze the remaining integrations over ξ_is

$$\Sigma^{(2n)} = 2d \frac{(-i)^{2n+1}(Ja^d)^{2n}\Gamma[n(2-2d)-1]}{(n-2)![-i(\omega-k_y)]^{n(2-2d)-1}} I_n(\alpha), \qquad (22)$$

where

$$I_n(\alpha) = \int_0^\infty \prod_{i=1}^n d\xi_i \frac{\xi_1^{1-2d}\xi_2^{-2d}\cdots\xi_{n-1}^{-2d}\xi_n^{1-2d}}{(\alpha+\xi_1+\xi_2+\cdots+\xi_n)^{n(2-2d)-1}}. \qquad (23)$$

We notice that for $n=2,3$ the integral in Eq. (23) diverges at the upper limit. In this case the expansion in Eq. (21) is not justified. These values of n have to be treated separately, (see Appendix A for details). For $n=2$ the most singular part is given in Eq. (13) and for $n=3$ we obtain, (see Appendix A 2).

$$\Sigma^{(6)} = 2d \frac{-i(Ja^d)^6 \Gamma(1-2d)\Gamma^2(2-2d)}{[-i(\omega-k_y)]^{5-6d}} \log \alpha. \qquad (24)$$

For $n \geq 3$ the integral in Eq. (23) is

$$I_n(\alpha) = (1-2d)^2 \alpha^{3-n} \frac{\Gamma^n(1-2d)\Gamma(n-3)}{\Gamma[n(2-2d)-1]}. \qquad (25)$$

The contributions of the orders $n \geq 3$ give

$$\sum_{n\geq 3} \Sigma^{(2n)} = 2d\alpha \frac{-i(Ja^d)^4 \Gamma^2(2-2d)}{[-i(\omega-k_y)]^{3-4d}}$$

$$\times \log(1+x)2d \frac{-i(Ja^d)^6 \Gamma(1-2d)\Gamma^2(2-2d)}{[-i(\omega-k_y)]^{5-6d}}$$

$$\times [1 - \log(1+x)], \qquad (26)$$

where

$$x = \frac{\alpha^{-1}(Ja^d)^2 \Gamma(1-2d)}{[-i(\omega-k_y)]^{(2-2d)}}. \qquad (27)$$

The sum of contributions in Eqs. (12), (13), (24), and (26) yields the final result Eq. (7).

IV. GREEN'S FUNCTION AT THE SHADOW MASS SHELL, $\omega \sim k_y$

In this section we turn to the analysis of the behavior of the Green's function at the "shadow side" of the pocket, $\omega \sim k_y$. As this region is separated from the mass shell, instead of the self-energy it is more convenient to study the Green's function itself. It is also convenient to discuss the amputated propagator, $\bar{\Sigma}_\omega(\mathbf{k}) = G_\omega(\mathbf{k})[G_\omega^{(0)}(\mathbf{k})]^{-2}$. To the second order in J, $\bar{\Sigma}_\omega(\mathbf{k})$ is given by Eq. (12), and is singular at $\omega = k_y$. At the next, fourth order, the correction (see Appendix B) has stronger singularity

$$\bar{\Sigma}^{(4)} \approx \frac{i(Ja^d)^4 \Gamma(1-2d)\Gamma(2-2d)}{[-i(\omega-k_x)]^{3-4d}} \alpha^{2-2d}. \qquad (28)$$

In what follows we perform a resumation the most singular terms in the expansion of $\bar{\Sigma}$ to all orders in J. Here, similarly to the analysis in Sec. III the solution is based on the observation that the off-mass shell particle is almost immediately scattered by a fluctuation and propagates large distance in Y

direction afterward. In other words, due to the condition $\omega \sim k_y$, the vertical segments of the real-space trajectory, see Fig. 3(b) are longer than the horizontal segments by a factor of $(\omega - k_x)/(\omega - k_y)$.

Keeping the above considerations in mind, for $n \geq 2$ we introduce new variables

$$Y\xi_1 = x_1 - 0, \quad Y\xi_2 = x_2 - x_1, \ldots, Y\xi_{n-1} = X - x_{n-2}. \quad (29)$$

Integrating over y_is variables we write the correction of order $(2n)$ to the amputated Green's function as

$$\bar{\Sigma}^{(2n)} = \frac{(-i)^{2n-1}(Ja^d)^{2n}}{[-i(\omega - k_x)]^{2n(1-d)-1}} \frac{\Gamma[2n(1-d)-1]}{(n-1)!} \bar{I}_n(\alpha), \quad (30)$$

where the remaining integrals

$$\bar{I}_n(\alpha) = \prod_{i=1}^{n-1} \int_0^\infty d\xi_i \frac{\xi_1^{-2d} \cdots \xi_{n-1}^{-2d}}{(\alpha^{-1} + \xi_1 + \cdots + \xi_{n-1})^{2n(1-d)-1}} \quad (31)$$

are convergent at the upper limit and can be evaluated as

$$\bar{I}_n(\alpha) = \alpha^{n-2d} \frac{\Gamma^{n-1}(1-2d)\Gamma(n-2d)}{\Gamma(2n(1-d)-1)}. \quad (32)$$

As a result we obtain for the singular part

$$\bar{\Sigma}^{(2n)} = \frac{i(-iJa^d)^{2n}\alpha^{n-2d}\Gamma^{n-1}(1-2d)\Gamma(n-2d)}{(n-1)![-i(\omega - k_x)]^{2n(1-d)-1}}. \quad (33)$$

For $n = 1$ the last expression reduces to the second-order corrections, Eq. (12). The sum of singular contributions Eq. (33) yields the result of Eq. (9).

V. CONCLUSIONS

First we would like to make a remark about the easy-axis anisotropy regime where the phase transition is in the Ising-model universality class. From our calculations it is easy to see that as far as the singular terms are concerned, the results remain unchanged provided one considers only one particular value for the scaling dimension: $d = 1/8$. This is despite the fact that multipoint correlation functions of the Ising-model order parameter fields are more complicated than Eq. (19). However, the singularities are determined by more simple correlators, namely, by the diagrams where pairs of the operators are very close to each other (see Fig. 3) resulting in a fusion of two order parameter fields. In the Ising model such fusion generates the energy-density operator and in the XY model it generates the gradient of ϕ field. Both operators have the same multipoint correlation functions.

Now we can discuss the results. They are well illustrated by Figs. 1, 2, and 4. The region of applicability of our calculations involves the energy scale $T_K = J(Ja)^{d/(d-1)}$. The result at the mass shell is valid for $|k_y| > T_K$ and the result at the shadow mass shell holds at $|k_x| > T_K$. With increasing temperature the spectral weight is transferred toward the bare Fermi surface and the shadow band feature quickly fades away as is clearly seen on Fig. 4 where the darker areas correspond to large values of the spectral density.
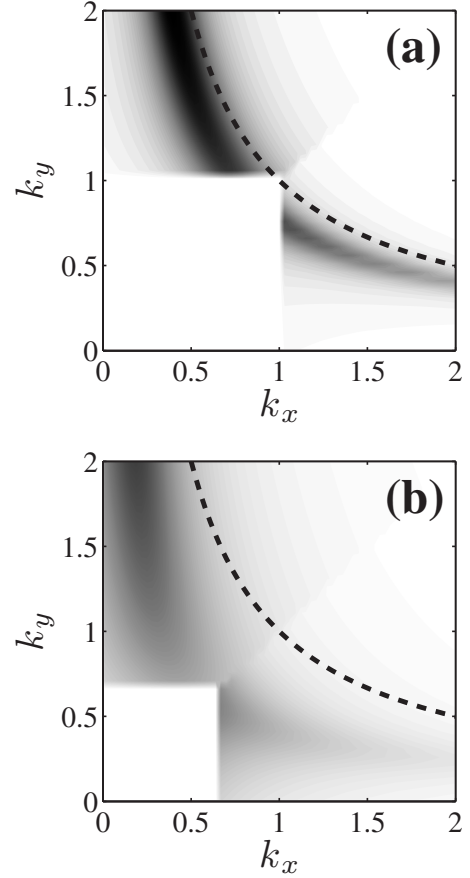

FIG. 4. False color plot representing the spectral density at the Fermi surface, $A_{\omega=0}(\mathbf{k})$, as given by Eqs. (7) and (9) at $k_y > k_x$ and $k_x > k_y$, respectively. The region $k_x, k_y < T_K$ where the results are inapplicable is not shown. The parameter d is (a) $d = 0.04$ and (b) $d = 0.13$. Dashed line shows the mean-field Fermi surface as given by Eq. (4).

Although our model does not include all the features ascribed to the cuprates, the results obtained may serve as a good qualitative guide to the problem. For instance, we see that critical thermal fluctuations give rise to the characteristic linear temperature dependence $\sim T$ of the spectral peak width, see Ref. 7 for alternative approach. This is an indication that such fluctuations are responsible of this feature in the cuprates. Our results demonstrate that with the rise of temperature the renormalized mass shell identified as the maximum intensity line in Fig. 4 approaches the bare Fermi surface while the peak becomes rather incoherent. The backside of the Fermi pockets fades away so that the pockets now look like arcs. These effects are qualitatively similar to that of the quantum fluctuations studied in Ref. 6 though the intensity of quantum fluctuations is regulated not by temperature but by the interactions.

In summary, we have presented systematic study of the thermal-fluctuations effects in two-dimensional system of electrons in interaction with SDW order parameter. In particular, the spectral density has been found to be strongly sensitive to these fluctuations. Fluctuations tend to restore the noninteracting Fermi-surface topology thus overriding the effects of the SDW order.

APPENDIX A: PERTURBATION THEORY AT THE MASS SHELL

In the present appendix we evaluate the most singular corrections to the self-energy at the mass shell, $\omega \sim k_x$.

1. Fourth-order contributions

In the fourth order the general expression in Eq. (16) takes the form

$$\Sigma_\omega^{(4)}(\boldsymbol{k}) = iJ^4 \int_0^\infty dx_2 e^{i(\omega - k_x)x_2} \int_0^\infty dy_2 e^{i(\omega - k_y)y_2} \int_0^{y_2} dy_1$$
$$\times C^{(4)}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{p}_1, \boldsymbol{p}_2), \tag{A1}$$

where $\boldsymbol{r}_1 = (0,0)$, $\boldsymbol{r}_2 = (x_2, y_1)$, $\boldsymbol{p}_1 = (0, y_1)$, and $\boldsymbol{p}_2 = (x_2, y_2)$ (see Fig. 3). Equation (21) gives

$$C^{(4)} = \frac{a^{4d}}{y_1^{2d}(y_2 - y_1)^{2d}} \left\{ \frac{[(y_2 - y_1)^2 + x_2^2]^d (y_1^2 + x_2^2)^d}{(x_2^2 + y_2^2)^d x_2^{2d}} - 1 \right\}. \tag{A2}$$

We expect the main contribution to come from the region $y_1, y_2 - y_1 \ll x_2$ it is convenient to introduce new variables, $y_2 = \xi x_2$ and $y_1 = \eta \xi x_2$. To isolate the leading singularity in Eq. (A1) we expand the cumulant in Eq. (A2) for $\xi \lesssim 1$

$$C^{(4)} \approx -2d a^{4d} \xi^{2-4d} \eta^{1-2d} (1 - \eta)^{1-2d}. \tag{A3}$$

This expansion holds for any $d < 1/2$.

$$\Sigma^{(4)} = -2di(Ja^d)^4 \Gamma(3 - 4d)$$
$$\times \int_0^\infty d\xi \int_0^1 d\eta \frac{\xi^{3-4d} \eta^{1-2d}(1 - \eta)^{1-2d}}{\{-i[(\omega - k_y)\xi + (\omega - k_x)]\}^{3-4d}}. \tag{A4}$$

The integral in Eq. (A4) is not convergent at the upper limit. This is an artifact of the approximation in Eq. (A3) which is not justified for $\xi \gtrsim 1$. To overcome this we differentiate Eq. (A1) twice with respect to the parameter α defined by Eq. (14)

$$\frac{\partial^2 \Sigma^{(4)}}{\partial \alpha^2} = -2di\Gamma(5 - 4d)(Ja^d)^4 [-i(\omega - k_y)]^{4d-3}$$
$$\times \int_0^\infty d\xi \int_0^1 d\eta \frac{\xi^{3-4d} \eta^{1-2d}(1 - \eta)^{1-2d}}{(\xi + \alpha)^{5-4d}}. \tag{A5}$$

The integrations in Eq. (A5) are easily done with the result

$$\frac{\partial^2 \Sigma^{(4)}}{\partial \alpha^2} = -2di\alpha^{-1}\Gamma^2(2 - 2d)\frac{(Ja^d)^4}{[-i(\omega - k_y)]^{3-4d}}. \tag{A6}$$

Integrating Eq. (A6) back we finally get

$$\Sigma^{(4)} = -2di\Gamma^2(2 - 2d)(Ja^d)^4 [-i(\omega - k_y)]^{4d-3}$$
$$\times (\alpha \log \alpha + A\alpha + B) \tag{A7}$$

with A and B constants. In the last equation the most singular term is presented in Eq. (13).

2. Order J^6

In this case the general expression (16) with $n = 3$ reduces to

$$\Sigma_\omega^{(6)}(\boldsymbol{k}) = i(-iJa^d)^6 \int_0^\infty dx_3 e^{i(\omega - k_x)x_3} \int_0^\infty dy_3 e^{i(\omega - k_y)y_3}$$
$$\times \int_0^{x_3} dx_2 \int_0^{y_3} dy_2 \int_0^{y_2} dy_1$$
$$\times \frac{ABC - B - C + 1}{y_1^{2d}(y_2 - y_1)^{2d}(y_3 - y_2)^{2d}}, \tag{A8}$$

where

$$A = \frac{(x_3^2 + y_2^2)^d |x_3^2 + (y_3 - y_1)^2|^d}{[x_3^2 + (y_2 - y_1)^2]^d (x_3^2 + y_3^2)^d}, \tag{A9}$$

$$B = \frac{[(x_3 - x_1)^2 + (y_2 - y_1)^2]^d}{(x_3 - x_1)^{2d}} \times \frac{[(x_3 - x_1)^2 + (y_3 - y_2)^2]^d}{[(y_3 - y_1)^2 + (x_3 - x_1)^2]^d} \tag{A10}$$

and

$$C = \frac{|x_1^2 + y_1^2|^d |x_1^2 + (y_2 - y_1)^2|^d}{|x_1|^{2d} |y_2^2 + x_1^2|^d}. \tag{A11}$$

We write

$$ABC - B - C + 1 = (A - 1)BC + (B - 1)(C - 1). \tag{A12}$$

It is apparent form Eq. (A12) that the leading singularity originates from the first term, $(A - 1)BC \approx A - 1 \approx -2d(y_1 - 0)(y_3 - y_2)$. Introducing new variables as in the Sec. III and integrating over x_is we obtain

$$\Sigma^{(6)} = 2di\frac{(Ja^d)^6 \Gamma(5 - 6d)}{[-i(\omega - k_y)]^{5-6d}}$$
$$\times \int_0^\infty \prod_{i=1}^3 d\xi_i \frac{\xi_1^{1-2d} \xi_2^{-2d} \xi_3^{1-2d}}{(\alpha + \xi_1 + \xi_2 + \xi_3)^{5-6d}}. \tag{A13}$$

Here again, the integrals are divergent on the upper limit. Similarly to the previous section we differentiate it once with respect to the variable α introduced in Eq. (14) in order to isolate the leading logarithmic singularity

$$\frac{\partial \Sigma^{(6)}}{\partial \alpha} = -2di\alpha^{-1}\frac{(Ja^d)^6\Gamma(6-6d)}{[-i(\omega-k_y)]^{5-6d}}$$

$$\times \int_0^\infty \prod_{i=1}^3 d\xi_i \frac{\xi_1^{1-2d}\xi_2^{-2d}\xi_3^{1-2d}}{(1+\xi_1+\xi_2+\xi_3)^{6-6d}}. \quad \text{(A14)}$$

The remaining integrals are easily evaluated. The subsequent integration over α restores the singularity in the self-energy correction

$$\Sigma^{(6)} = 2d\frac{-i(Ja^d)^6\Gamma(1-2d)\Gamma^2(2-2d)}{[-i(\omega-k_y)]^{5-6d}}(\log \alpha + C),$$

$$\text{(A15)}$$

where C is an integration constant. The singular part of Eq. (A15) is given by Eq. (24).

APPENDIX B: LEADING SINGULARITIES AT THE SHADOW MASS SHELL, $\omega \sim k_y$ TO THE FOURTH ORDER

In this appendix we evaluate the singular contributions to the Green's function at the shadow mass shell, $\omega \sim k_y$ to fourth order in the coupling constant. We start with the expression (A1) introduced in Appendix A 1. In contrast to the discussion in Appendix A 1 we anticipate the singularity at $\omega = k_y$ to come from the region $y_2 \gg x_2$ and introduce new variables accordingly, $x_2 = \xi y_2$ and $y_1 = \eta y_2$. Performing integration over y_2 we obtain

$$\Sigma^{(4)} = i(Ja^d)^4\int_0^\infty d\xi \int_0^1 d\eta \frac{\Gamma(3-4d)\eta^{-2d}(1-\eta)^{-2d}}{\{-i[(\omega-k_y)+(\omega-k_x)\xi]\}^{3-4d}}$$

$$\times \left[\frac{|(1-\eta)^2+\xi^2|^d|\eta^2+\xi^2|^d}{|1+\xi^2|^d|\xi|^{2d}} - 1\right]. \quad \text{(B1)}$$

We notice that the singularity at $\omega \sim k_y$ comes from the region of small ξ. Therefore we keep only the first term in the square brackets in Eq. (B1). After performing remaining integrations over ξ_is we obtain

$$\Sigma^{(4)} = \frac{i(Ja^d)^4\Gamma(1-2d)\Gamma(2-2d)\alpha^{2-2d}}{[-i(\omega-k_x)]^{3-4d}}. \quad \text{(B2)}$$

We stress that contrary to the mass shell singularities discussed in Appendix A, where it was important to compute the self-energy, at the shadow mass shell it is enough to consider the Green's function itself.

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